JHEP HYPER VERSION

Received: *April 25, 2001*, Accepted: *June 6, 2001*

On the eleven-dimensional origins of polarized D0-branes

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Abstract: The worldvolume theory of a D0-brane contains a multiplet of fermions which can couple to background spacetime fields. This coupling implies that a D0-brane may possess multipole moments with respect to the various type-IIA supergravity fields. Different such polarization states of the D0-brane will thus generate different long-range supergravity fields, and the corresponding semi-classical supergravity solutions will have different geometries. In this paper, we reconsider such solutions from an eleven-dimensional perspective. We thus begin by deriving the "superpartners" of the eleven-dimensional graviton. These superpartners are obtained by acting on the purely bosonic solution with broken supersymmetries and, in theory, one can obtain the full BPS supermultiplet of states. When we dimensionally reduce a polarized supergraviton along its direction of motion, we recover a metric which describes a polarized D0-brane. On the other hand, if we compactify along the retarded null direction we obtain the short distance, or "near-horizon", geometry of a polarized D0-brane, which is related to finite N Matrix theory. The various dipole moments in this case can only be defined once the eleven-dimensional metric is "regularized" and, even then, they are formally infinite. We argue, however, that this is to be expected in such a non-asymptotically flat spacetime. Moreover, we find that the superpartners of the D0-brane, in this $r \to 0$ limit, possess neither spin nor D2-brane dipole moments.

Keywords: M-Theory, D-branes, M(atrix) Theories.

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1. Introduction

At present, it is unclear as to what are the fundamental physical degrees of freedom underlying M-theory. Most of what we understand about M-theory is based on the facts that it has eleven-dimensional supergravity as its low energy limit and, via compactification on a circle, can be related to type-IIA string theory [1, 2]. While the string of the type-IIA theory was at one time regarded as the fundamental object, it is now clear that this is not the basic degree of freedom underpinning M-theory as a whole. Of course, one may take the point of view that there is no *truly* fundamental physical degree of freedom but rather that, in different regions of the M-theory moduli space, different degrees of freedom appear.

An interesting approach to M-theory, which may resolve at least some of these conceptual issues, is Matrix theory: a non-relativistic supersymmetric quantum mechanics of $N \times N$ matrix degrees of freedom, as considered in an earlier guise in [3] (see, e.g., [4] for a recent review). Matrix theory was originally conjectured to be equivalent to M-theory in the infinite momentum frame, in the limit that N is large [5]. The *finite* N version has been further conjectured to be equivalent to the discrete light cone quantization (DLCQ) of M-theory [6, 7, 8]. The hamiltonian of Matrix theory is precisely the low-energy, or non-relativistic, limit of the hamiltonian describing a system of N type-IIA D0-branes (as such, the corresponding action can be obtained via the null reduction of the action describing the eleven-dimensional massless particle [9]). This makes sense given that the D0-brane couples to the Ramond-Ramond (R-R) vector of the type-IIA theory, which is itself the Kaluza-Klein vector obtained by dimensionally reducing M-theory on a circle. In other words, D0-branes are the "partonic", or fundamental, degrees of freedom underlying Matrix theory.

A key fact about D0-branes is that they possess "internal" degrees of freedom, which couple to spacetime fields. More specifically, on the world-volume of a D0-brane there resides a simple quantum mechanical theory which includes 16 fermionic operators θ. They generate an SO(16) Clifford algebra, so may be written as $2^{16/2} = 256$-dimensional gamma matrices [10], i.e. a D0-brane has 256 internal degrees of freedom, or polarization states.[1] This space of states has been constructed in [10], and is just the space of polarization states of the supergraviton in eleven dimensions. In the weak field approximation, the worldvolume fermions couple to small fluctuations of the background metric, h_{ab}, NS-NS 2-form potential, B_{ab}, and R-R 1-form and 3-form potentials, C_a and C_{abc}, via the terms [16, 17, 10]

$$\mathcal{L}_{D0} = -\frac{i}{8}\left(\partial_i h_{tj} + \partial_i C_j\right)\bar{\theta}\gamma^{ij}\theta + \frac{i}{16}\left(\partial_i B_{jk} + \partial_i C_{tjk}\right)\bar{\theta}\gamma^{ijk}\theta\,. \tag{1.1}$$

In other words, the internal degrees of freedom generate non-trivial long-range supergravity fields.

1.1 Bosonic p-branes and their superpartners

The D0-brane solution of type-IIA supergravity is an example of a more general class of extremal p-brane. These solitonic solutions of ten- and eleven-dimensional supergravity have been much studied (see, e.g., [18, 19] for reviews). One is usually interested in purely bosonic solitons, which nevertheless admit Killing spinors, such solutions thus being invariant under some fraction of the 32 supersymmetries. For a single p-brane, this fraction is one half, which leaves 16 *broken* supersymmetries. They correspond to 16 zero mode fermions, the presence of which gives rise to the entire BPS supermultiplet of 256 states. These polarization states of the spinning p-brane fall into representations of the little group of the respective brane so, as we have already mentioned, the states of the spinning D0-brane match those of the eleven-dimensional supergraviton; the little group in both cases is SO(9).

[1] D0-branes in different polarization states have different interactions: in addition to the usual velocity-dependent terms, there are also spin-dependent static forces, which were first studied in [11, 12]. Other studies in the context of Matrix theory include [13, 14, 15].

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From the worldvolume perspective, the polarization state of a spinning p-brane is reflected in its couplings to the background supergravity fields, via terms such as (1.1). The generic state couples to *all* such fields, not just the metric, $(p+1)$-form potential and dilaton. As can be seen from (1.1), certain states of the D0-brane, for example, have dipole moments with respect to the R-R 3-form potential [10], in addition to moments with respect to the other bosonic fields.[2] We should further note that the various quadrupole couplings and moments have also been worked out in [16].

Each such polarization state has a corresponding supergravity solution, which displays the multipole moments in question. These are the so-called "superpartners" [21] of the purely bosonic solutions, and they can be generated by acting on the latter with broken supersymmetry transformations, as first discussed by Aichelburg and Embacher [21]. They exhibited the complete supermultiplet containing the extreme Reissner-Nordström multiple black hole solutions in four-dimensional $\mathcal{N}=2$ supergravity. One considers the finite transformation

$$\mathbf{\Phi} \to \mathbf{\Phi}' = e^{\delta_\epsilon}\mathbf{\Phi} = \mathbf{\Phi} + \delta_\epsilon \mathbf{\Phi} + \frac{1}{2}\delta_\epsilon^2 \mathbf{\Phi} + \cdots , \tag{1.2}$$

where $\mathbf{\Phi}$ denotes the bosonic solution, and δ_ϵ the action of a broken infinitesimal supersymmetry transformation with parameter ϵ. Of course, a single such transformation leaves the field equations invariant at the linearized level only, but the finite transformation (1.2) will be a symmetry of the full non-linear field equations.

The first order variation, δ_ϵ, generates fermionic "hair", and so a non-vanishing supercharge. Corrections to the bosonic fields, which give rise to various dipole moments, are generated by the second order variations, δ_ϵ^2, and so on. Since ϵ is a Grassman quantity, the series (1.2) terminates at δ_ϵ^{16} for the ten- and eleven-dimensional supergravities; ϵ has 16 independent components in these cases. Complete supermultiplets containing the bosonic p-brane solutions of these theories are thus unlikely to be found, but spin and magnetic dipole moments are readily analysed; in addition to black hole and string states in four-dimensional $\mathcal{N}=4$ string compactifications [22], superpartners of the D0-brane [23, 10], the M2-brane [24] and the M5-brane [25] have been studied using such techniques.

As localized solitons, p-branes possess bosonic zero modes corresponding to broken translational symmetries, the associated collective coordinates specifying the centre-of-mass position of the brane in question. In a similar manner, the 16 broken supersymmetries correspond to fermionic zero modes so, in a sense, these fermionic moduli can be thought of as fermionic "collective coordinates". There is an important difference, however [26, 27, 24]: the fermionic zero modes satisfy non-trivial

[2] It should be emphasized, however, that such polarized branes are different in nature to the purely bosonic dielectric branes of Myers [20]. The dipole moments of the latter have their origin in the non-abelian worldvolume theory of multiple D-branes, and the resulting solutions are spatially extended. The polarization states considered herein, however, are states of a *single* brane.

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anti-commutation relations, inherited from the anti-commutation relations of the spacetime fermions. Since they must be realised as operators, the back-reaction of the fermionic zero modes on the supergravity fields results in operator-valued expressions, and the background fields have a meaning only in the sense of an expectation value for a given BPS state [24]. Moreover, the possible polarization states form representations of the algebra of fermionic zero mode operators, which is SO(16) in the ten- and eleven-dimensional examples. Given this space of states, one can then ask the question as to whether the original bosonic solution is *exact*, i.e. whether the back-reaction of the fermionic zero modes vanishes in the corresponding BPS state, which will be a singlet under the respective little group. Since the SO(16) vacuum state is an SO(8) singlet, the purely bosonic M2-brane is thus exact as a BPS state [24], although this is not the case for the D0-brane: there is no SO(9) singlet of SO(16), so the purely bosonic D0-brane soliton is not an exact BPS state [23]. Of course, the same can be said of the purely bosonic eleven-dimensional supergraviton.

It is precisely the background supergravity fields in (1.1) which are determined using these superpartner generating techniques. With respect to the D0-brane, rather than working with type-IIA supergravity directly, an alternative as suggested in [10] is to work with the eleven-dimensional gravitational, or pp-wave. Our purpose here is to consider this approach. We compute the superpartners of the purely bosonic pp-wave in the following section, and dimensionally reduce them in the standard way [28, 29] to give the superpartners of the D0-brane in section 3. We find exact agreement with the results of [23, 10]. Indeed, the fact that the gyromagnetic ratios of the D0-brane are both equal to 1 [23, 16, 10] is a natural consequence of their eleven-dimensional origins. Such gyromagnetic ratios are characteristic of Kaluza-Klein states [23]. Given the connections between the DLCQ of M-theory and Matrix theory at finite N, it is of further interest to consider the dimensional reduction on a light-like circle (or rather an asymptotically light-like circle). This we do in section 4, before concluding. It would seem that physical properties of the resulting ten-dimensional solution, which is not asymptotically flat, are most easily analysed from this eleven-dimensional perspective. Our conventions are given in an appendix.

2. Eleven-dimensional supergravitons

2.1 The bosonic solution

Eleven-dimensional supergravity [30] consists of the elfbein $E^{\underline{A}}_{\ A}$, a 3-form potential A_{ABC}, with 4-form field strength $F_{ABCD} = 4\partial_{[A}A_{BCD]}$, and a spin 3/2 Majorana gravitino Ψ_A. In our conventions, the lagrangian and equations of motion are invariant under the infinitesimal supersymmetry transformations

$$\begin{aligned} \delta_\epsilon E^{\underline{A}}_{\ A} &= i\,\bar\epsilon\,\Gamma^{\underline{A}}\Psi_A\,, \\ \delta_\epsilon A_{ABC} &= -3i\,\bar\epsilon\,\Gamma_{[AB}\Psi_{C]}\,, \end{aligned}$$

$$\delta_\epsilon \Psi_A = \mathcal{D}_A(\tilde{\omega})\,\epsilon\,, \tag{2.1}$$

where ϵ is an arbitrary anticommuting Majorana spinor and

$$\begin{aligned}
\mathcal{D}_A(\tilde{\omega}) &= \partial_A + \frac{1}{4}\tilde{\omega}_{A\underline{A}\underline{B}}\Gamma^{\underline{A}\underline{B}} - \frac{1}{288}\left(\Gamma_A{}^{BCDE} - 8\delta_A^B\Gamma^{CDE}\right)\mathcal{F}_{BCDE}\,,\\
\tilde{\omega}_{A\underline{A}\underline{B}} &= \omega_{A\underline{A}\underline{B}} - \frac{i}{2}\left(\bar{\Psi}_A\Gamma_{\underline{B}}\Psi_{\underline{A}} - \bar{\Psi}_A\Gamma_{\underline{A}}\Psi_{\underline{B}} + \bar{\Psi}_{\underline{B}}\Gamma_A\Psi_{\underline{A}}\right),\\
\mathcal{F}_{ABCD} &= F_{ABCD} - 3i\bar{\Psi}_{[A}\Gamma_{BC}\Psi_{D]}\,.
\end{aligned} \tag{2.2}$$

$\omega_{A\underline{A}\underline{B}}$ denotes the standard spin connection. We also make use of the infinitesimal SO$(10,1)$ Lorentz transformations which, in terms of the arbitrary infinitesimal parameter $\Lambda_{\underline{A}\underline{B}} = -\Lambda_{\underline{B}\underline{A}}$, are

$$\begin{aligned}
\delta_L E_A^{\underline{A}} &= \Lambda^{\underline{A}}{}_{\underline{B}} E_A^{\underline{B}}\,,\\
\delta_L \Psi_A &= \frac{1}{4}\Lambda_{\underline{A}\underline{B}}\Gamma^{\underline{A}\underline{B}}\Psi_A\,.\\
\delta_L A_{ABC} &= 0\,.
\end{aligned} \tag{2.3}$$

Switching off the 3-form and gravitino, we have pure eleven-dimensional gravity, the equations of motion of which admit solutions with a null Killing vector, as first discussed by Hull [31]. They describe gravitational waves propagating at the speed of light. Denoting the direction of propagation by z, the metric can be written as

$$ds^2 = -(2-H)dt^2 + Hdz^2 + 2(1-H)dzdt + dx^i dx^i\,, \tag{2.4}$$

where $x^i, i = 1,\ldots,9$ denote the cartesian coordinates on $\mathbb{R}^9$, the space transverse to the tz-plane. The function H is harmonic on this space, the asymptotically flat solution being given by

$$H(r) = 1 + \frac{2}{7}\frac{\kappa_{11}^2}{\Omega_8}\frac{P}{r^7}\,, \tag{2.5}$$

where $r = |x|$ is the radial coordinate on $\mathbb{R}^9$ and Ω_8 is the volume of a unit eight-sphere. In general, the momentum density, P, has a dependence on the retarded time, $x^- = t - z$, so that the amplitude of the pp-wave varies across the wave-front. The ADM energy-momentum is then

$$P^t = P^z = \int dz P(x^-)\,. \tag{2.6}$$

As we are ultimately interested in the dimensional reduction of the pp-wave in the z direction, we assume $P = P^z/2\pi R = \text{const.}$, with R the compactification radius. In this case, the solution describes a plane-fronted parallel gravitational wave (pp-wave for short). Making use of the null Killing direction, one can also find pp-wave solutions with non-vanishing 3-form and gravitino [31]. But as these are *not* given by acting on the purely bosonic solution (2.4) with supersymmetry transformations [31], they are not the type of solution we are looking for.

Of course, the pp-wave (2.4) can be viewed as the extremal limit of an infinitely boosted Schwarzschild black hole, or rather of an infinitely boosted uncharged black string [32, 33]. To see this, take

$$ds^2 = -\left(1-\frac{M}{r^7}\right)dt'^2 + dz'^2 + \left(1-\frac{M}{r^7}\right)^{-1}dr^2 + r^2 d\Omega_8^2\,, \tag{2.7}$$

and perform the boost

$$t' = \cosh\mu\, t - \sinh\mu\, z\,, \qquad z' = \cosh\mu\, z - \sinh\mu\, t\,. \tag{2.8}$$

One finds

$$ds^2 = -\left(1-\frac{M}{r^7}\right)H^{-1}dt^2 + H\left(dz + \coth\mu(H^{-1}-1)dt\right)^2 + \left(1-\frac{M}{r^7}\right)^{-1}r^2 + r^2 d\Omega_8^2\,, \tag{2.9}$$

where

$$H(r) = 1 + \sinh^2\mu\frac{M}{r^7}\,, \tag{2.10}$$

is the harmonic function associated with the wave. It carries momentum density $P \sim \sinh^2\mu\, M$. Keeping P fixed whilst taking the extremal limit, $M \to 0$, requires an infinite boost, $\mu \to \infty$. In this limit, the metric (2.9) takes the form (2.4) as promised.

2.2 Superpartners

We analyse the pp-wave (2.4) in the pseudo-orthonormal basis:

$$E^{\underline{t}} = H^{-1/2}dt, \qquad E^{\underline{z}} = H^{-1/2}(1-H)dt + H^{1/2}dz, \qquad E^{\underline{i}} = dx^i\,, \tag{2.11}$$

where

$$ds^2 = -(E^{\underline{t}})^2 + (E^{\underline{z}})^2 + E^{\underline{i}}E^{\underline{i}}\,, \tag{2.12}$$

and the triangular parameterization, $E^{\underline{t}}{}_z = 0 = E^{\underline{i}}{}_z$, allows for the dimensional reduction in the z direction. The superpartners of the bosonic wave are generated by acting on the solution (2.11) with the supersymmetry transformations (2.1). As explained in [28, 29], to restore the triangular form of the elfbein, and to ensure the canonical form of the ten-dimensional supersymmetry transformations, we must perform compensating $\mathrm{SO}(10,1)$ and $\mathrm{SO}(9,1)$ Lorentz transformations. Schematically [28],

$$\delta_\eta(D=10) = \delta_\epsilon(D=11) + \delta_{L_1}(\Lambda^{\underline{a}}{}_{\underline{z}}) + \delta_{L_2}(\Lambda^{\underline{a}}{}_{\underline{b}})\,, \tag{2.13}$$

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where the ten-dimensional supersymmetry parameter, η, is related to ϵ as will become clear. We thus consider the overall transformation

$$
\begin{aligned}
\delta_\epsilon E_A^{\underline{A}} &= (\Lambda_1{}^{\underline{A}}{}_{\underline{B}} + \Lambda_2{}^{\underline{A}}{}_{\underline{B}}) E_A^{\underline{B}} + i\,\bar\epsilon\,\Gamma^{\underline{A}}\Psi_A\,,\\
\delta_\epsilon \Psi_A &= \frac{1}{4}(\Lambda_{1\,\underline{A}\,\underline{B}} + \Lambda_{2\,\underline{A}\,\underline{B}})\Gamma^{\underline{A}\,\underline{B}} + \mathcal{D}_A(\tilde\omega)\epsilon\,,\\
\delta_\epsilon A_{ABC} &= -3i\,\bar\epsilon\,\Gamma_{[AB}\Psi_{C]}\,,
\end{aligned}
\tag{2.14}
$$

where [28, 29]

$$
\begin{aligned}
\Lambda_1{}^{\underline{a}}{}_{\underline{z}} &= -\,i\bar\epsilon\,\Gamma^{\underline{a}}\Psi_{\underline{z}}\,,\\
\Lambda_2{}^{\underline{a}}{}_{\underline{b}} &= \frac{i}{8}\,\bar\epsilon\,\Gamma^{\underline{a}}{}_{\underline{b}}\,\Gamma^{\underline{z}}\,\Psi_{\underline{z}}\,.
\end{aligned}
\tag{2.15}
$$

The resulting fields will then be given by (1.2) with δ_ϵ as in (2.14).

Since the original solution (2.4) has $\Psi_A = 0$, a single transformation generates fermions alone. We have

$$
\begin{aligned}
\delta_\epsilon \Psi_t &= \partial_t \epsilon - \frac{1}{2}H^{-1/2}\partial_i H\,\Gamma^{\underline{i}\,\underline{t}}P_+\epsilon\,,\\
\delta_\epsilon \Psi_z &= \partial_z \epsilon + \frac{1}{2}H^{-1/2}\partial_i H\,\Gamma^{\underline{i}\,\underline{t}}P_+\epsilon\,,\\
\delta_\epsilon \Psi_i &= \partial_i \epsilon + \frac{1}{4}H^{-1}\partial_i H\epsilon - \frac{1}{2}H^{-1}\partial_i H P_+\epsilon\,,
\end{aligned}
\tag{2.16}
$$

where the projection operators

$$
P_\pm = \frac{1}{2}\left(1 \pm \Gamma^{\underline{t}}\Gamma^{\underline{z}}\right), \tag{2.17}
$$

can be used to split the supersymmetry parameter as $\epsilon = P_+\epsilon + P_-\epsilon \equiv \epsilon_+ + \epsilon_-$. The unbroken supersymmetry parameters are given by $\epsilon = H^{-1/4}\epsilon_-$, so that $P_+\epsilon = 0$ and the bosonic solution (2.4) does indeed preserve one half of the supersymmetries. It is then clear that any spinor $\epsilon = \mathcal{E}(x^i)\epsilon_0$, for $\mathcal{E} \neq H^{-1/4}$ or $\epsilon_0 \neq \epsilon_-$ will generate a gravitino. However, if the associated supercharge is to be non-zero and finite, we should choose $\epsilon_0 = \epsilon_+$, and demand that $\mathcal{E} \to 1$ as $r \to \infty$ [21].

The choice of function $\mathcal{E}$ is then a choice of gauge. One way to fix the gauge freedom is to impose the tracelessness condition, $\Gamma^A\delta_\epsilon\Psi_A = 0$, on the first order gravitino [34], so that it is a pure spin 3/2 excitation. On the other hand, perhaps a more fundamental criterion is that the first order gravitino be normalizable [25]. In many cases, these two restrictions coincide [34], but it is not clear whether they will in general [25] (see [35] for a discussion of these issues). In the case at hand, we take $\mathcal{E} = H^{-1/4}$ in analogy with the unbroken supersymmetries. Substituting for $\epsilon = H^{-1/4}\epsilon_+$ in (2.16) gives

$$
\delta_\epsilon \Psi_t = -\frac{1}{2}H^{-3/4}\partial_i H\,\Gamma^{\underline{i}\,\underline{t}}\,\epsilon_+\,,
$$

$$
\begin{aligned}
\delta_\epsilon \Psi_z &= \frac{1}{2} H^{-3/4} \partial_i H \, \Gamma^{\underline{i}\,\underline{t}} \, \epsilon_+ \, , \\
\delta_\epsilon \Psi_i &= -\frac{1}{2} H^{-5/4} \partial_i H \, \epsilon_+ \, .
\end{aligned} \tag{2.18}
$$

It is easy to check that this first order gravitino is normalizable:

$$
|\delta_\epsilon \Psi|^2 = \frac{1}{\kappa_{11}^2} \int_\Sigma d^{10}x \sqrt{g_{(10)}} \Psi_A^\dagger \Psi_B g^{AB} = \frac{1}{2} P^z \epsilon_+^\dagger \epsilon_+ \, , \tag{2.19}
$$

where Σ denotes a space-like hypersurface with induced metric $g_{(10)}$. This we take as justification for our choice of $\mathcal{E}$ and ϵ_0. However, the tracelessness condition, $\Gamma^A \delta_\epsilon \Psi_A = 0$, is *not* satisfied for this choice.

With these first order variations, a second application of the transformations (2.14) generates further bosonic fields, given in terms of fermion bilinears. The second order variation of the elfbein is

$$
\begin{aligned}
\delta_\epsilon^2 E^{\underline{t}}{}_i &= \frac{i}{16} H^{-3/2} \partial_j H \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ \, , \\
\delta_\epsilon^2 E^{\underline{i}}{}_t &= -\frac{7i}{16} H^{-2} \partial_j H \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ \, , \\
\delta_\epsilon^2 E^{\underline{z}}{}_i &= \frac{i}{2} H^{-3/2} \partial_j H \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ \, ,
\end{aligned} \tag{2.20}
$$

which, up to $\mathcal{O}(\epsilon^4)$ terms, gives the metric

$$
ds^2 = -(2-H)dt^2 + Hdz^2 + 2(1-H)dzdt - \frac{i}{2} H^{-1} \partial_j H \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ (dt - dz) dx^i + dx^i dx^i \, . \tag{2.21}
$$

The fermion bilinears in the metric (2.21), and in the 3-form derived below, can be expressed in terms of SO(9) creation and annihilation operators [10]; can one see explicitly that these fields have a meaning only in the sense of an expectation value for a given BPS state.

Since $\delta_\epsilon^2 g_{ij} = 0$, and since $\delta_\epsilon^2 g_{ti}$ falls off too quickly to affect the relevant integrals at infinity, the fields (2.20) do not alter the ADM energy-momentum and all members of the supermultiplet are massless. However, the $g_{zi} = -g_{ti}$ off-diagonal components of the metric give rise to a conserved angular momentum, $J_{\mu ij}$, where we denote the "worldvolume" directions by $\mu = t, z$. As in [24], the angular momentum carries a worldvolume index as well as a pair of transverse indices indicating the plane of rotation. It can easily be read off from the metric: with [36]

$$
g_{\mu i} \to -\frac{\kappa_{11}^2}{\Omega_8} J_{\mu ij} \frac{\hat{x}^j}{r^8} \, , \tag{2.22}
$$

as $r \to \infty$, we have

$$
J_{\mu ij} = -\frac{i}{2} P \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{\mu}} \, \epsilon_+ \, , \tag{2.23}
$$

which is an angular momentum density. Note that $g_{zi} = -g_{ti}$ gives $J_{zij} = -J_{tij}$. Being generated by a static fermion bilinear, the angular momentum is not of the Kerr-type and is more rightly interpreted as an "intrinsic" angular momentum or *spin* [21]. Upon dimensional reduction in the z-direction, J_{tij} and J_{zij} are respectively identified as the spin and magnetic dipole moments of the D0-brane.

Turning to the 3-form potential, which does not change under the Lorentz transformations, we find

$$\delta^2_\epsilon A_{\mu ij} = \frac{i}{2} H^{-1}\partial_k H\, \bar\epsilon_+\, \Gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{\mu}}\epsilon_+ \,. \tag{2.24}$$

All states of the supermultiplet thus have the BPS property $M = Q = 0$, since none are charged with respect to the 3-form: the fields (2.24) die off too quickly to affect integrals of the field strength at infinity. There is, however, a non-vanishing dipole moment associated with the 3-form potential (2.24). This is a 4-index tensor, $\mu_{\mu ijk}$, with a single worldvolume and three transverse indices. With [24]

$$A_{\mu ij} \to \frac{\kappa_{11}^2}{\Omega_8}\mu_{\mu ijk}\frac{\hat{x}^k}{r^8}\,, \tag{2.25}$$

as $r \to \infty$, we have

$$\mu_{\mu ijk} = -\frac{i}{2} P\, \epsilon_+ \Gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{\mu}}\, \epsilon_+ \,, \tag{2.26}$$

so that $\mu_{tijk} = -\mu_{zijk}$, as for the angular momentum (2.23). In ten dimensions, the dipole moments μ_{tijk} and μ_{zijk} are respectively identified as an electric dipole moment with respect to the R-R 3-form potential, and a magnetic dipole moment with respect to the NS-NS B field.

The superpartners also carry a supercharge density, given by the surface integral [37]

$$Q = \frac{i}{2\kappa_{11}^2}\oint_\infty dS_i\, \Gamma^{i\alpha}\, \Psi_\alpha\,, \tag{2.27}$$

where $\alpha = i, z$ runs over all *spatial* directions, and the gravitino is given by (2.18). We find

$$Q = -\frac{i}{2} P\, \epsilon_+ \,. \tag{2.28}$$

Higher-order corrections to the gravitino will not alter this value of $\bar\epsilon Q$.

3. Polarized D0-branes

To dimensionally reduce in the z-direction, we make the usual Kaluza-Klein ansatz for the elfbein:

$$E_A^{\ \underline{A}} = \begin{pmatrix} E_a^{\ \underline{a}} & E_a^{\ \underline{z}} \\ 0 & E_z^{\ \underline{z}} \end{pmatrix} = \begin{pmatrix} e^{-\phi/12} e_a^{\ \underline{a}} & e^{2\phi/3} C_a \\ 0 & e^{2\phi/3} \end{pmatrix}, \tag{3.1}$$

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with inverse

$$E^A{}_{\underline{A}} = \begin{pmatrix} E^a{}_{\underline{a}} & E^z{}_{\underline{a}} \\ 0 & E^z{}_{\underline{z}} \end{pmatrix} = \begin{pmatrix} e^{\phi/12} e^a{}_{\underline{a}} & -e^{\phi/12} C_{\underline{a}} \\ 0 & e^{-2\phi/3} \end{pmatrix} . \tag{3.2}$$

The 3-form potential reduces to the ten-dimensional R-R 3-form potential, C_{abc}, and NS-NS 2-form potential B_{ab}:

$$A_{ABC} = (A_{abc}, A_{abz}) = (C_{abc}, B_{ab}) . \tag{3.3}$$

It is important to note that the above fields are those of the full superpartner solutions. Take, for example, the $E^{\underline{z}}{}_z = e^{2\phi/3}$ component of the elfbein. This is really the transformed field

$$E'^{\underline{z}}{}_z = E^{\underline{z}}{}_z \left(1 + E^z{}_{\underline{z}} \delta_\epsilon E^{\underline{z}}{}_z + E^z{}_{\underline{z}} \frac{1}{2} \delta^2_\epsilon E^{\underline{z}}{}_z + \mathcal{O}(\epsilon^3) \right) , \tag{3.4}$$

which should be equated with

$$e^{2\phi'/3} = e^{2\phi/3} \left(1 + \frac{2}{3} \delta_\epsilon \phi + \frac{4}{9} \frac{1}{2} \delta^2_\epsilon \phi + \mathcal{O}(\epsilon^3) \right) , \tag{3.5}$$

so that $e^{2\phi} = H^{3/2}$ as required. However, since the $E^{\underline{z}}{}_z$ component of the elfbein receives no corrections, neither does the dilaton; and this simplifies matters considerably. As expected, the original pp-wave (2.4) gives the purely bosonic D0-brane solution

$$\begin{aligned} ds^2 &= -H^{-7/8} dt^2 + H^{1/8} dx^i dx^i , \\ C_t &= H^{-1} - 1 , \\ e^{2\phi} &= H^{3/2} . \end{aligned} \tag{3.6}$$

The longitudinal momentum is quantized and equal to the mass, T_0, of the D0-branes: $P^z = N/R = N/(g_s \sqrt{\alpha'}) = T_0$. Then the harmonic function H is

$$H(r) = 1 + 60\pi^3 g_s \frac{\alpha'^{7/2} N}{r^7} . \tag{3.7}$$

The first order variations (2.18) of Ψ_A reduce to corresponding variations of the ten-dimensional fermions. The ten-dimensional Dirac matrices are given by

$$\Gamma^{\underline{A}} = (\Gamma^{\underline{a}}, \Gamma^{\underline{z}}) = \left(\gamma^{\underline{a}}, \gamma^{11} \right) , \tag{3.8}$$

where $\gamma^{11} = \gamma^{\underline{t}} \ldots \gamma^{\underline{9}}$ is the ten-dimensional chirality operator, and the gravitino reduces in a similar manner:

$$\Psi_{\underline{A}} = (\Psi_{\underline{a}}, \Psi_{\underline{z}}) . \tag{3.9}$$

The ten-dimensional gravitino, $\psi_a = e^{\underline{a}}{}_a \psi_{\underline{a}}$, and dilatino, λ, are then given by [28, 29]

$$
\begin{aligned}
\Psi_{\underline{a}} &= e^{\phi/24}\left(\psi_{\underline{a}} - \frac{\sqrt{2}}{12}\gamma_{\underline{a}}\gamma^{11}\lambda\right), \\
\Psi_{\underline{z}} &= \frac{2\sqrt{2}}{3} e^{\phi/24}\lambda\,,
\end{aligned}
\tag{3.10}
$$

and the ten-dimensional supersymmetry parameter is $\eta = e^{\phi/24}\epsilon$ [28, 29]. This latter implies $\eta = H^{-7/32}\epsilon_+$ where, in ten-dimensional language

$$
P_+\epsilon_+ = \frac{1}{2}\left(1 + \gamma^{\underline{t}}\gamma^{11}\right)\epsilon_+ = \epsilon_+\,,
\tag{3.11}
$$

as should be expected [23, 10]. The variations of the ten-dimensional fermions are thus

$$
\begin{aligned}
\delta_\eta\lambda &= \frac{3\sqrt{2}}{8} H^{-41/32}\partial_i H\,\gamma^{\underline{i}\,\underline{t}}\,\epsilon_+\,, \\
\delta_\eta\psi_t &= -\frac{7}{16} H^{-55/32}\partial_i H\,\gamma^{\underline{i}\,\underline{t}}\,\epsilon_+\,, \\
\delta_\eta\psi_i &= \frac{1}{16} H^{-39/32}\partial_j H(\gamma^{\underline{i}\,\underline{j}} - 7\delta^{ij})\epsilon_+\,,
\end{aligned}
\tag{3.12}
$$

which agree precisely with the results of [23, 10]. Both the dilatino and gravitino are normalizable but, as in the eleven-dimensional case, the gravitino does not obey the condition $\gamma^a\delta_\eta\psi_a = 0$.

The second order corrections (2.20) and (2.24) of the elfbein and 3-form reduce respectively to second order variations of the zehnbein and R-R vector, and the R-R 3-form and NS-NS B field. We find

$$
\begin{aligned}
\delta^2_\eta e^{\underline{t}}{}_i &= \frac{i}{16} H^{-23/16}\partial_j H\,\bar\epsilon_+\,\gamma^{\underline{i}\,\underline{j}\,\underline{t}}\,\epsilon_+\,, \\
\delta^2_\eta e^{\underline{i}}{}_t &= -\frac{7i}{16} H^{-31/16}\partial_j H\,\bar\epsilon_+\,\gamma^{\underline{i}\,\underline{j}\,\underline{t}}\,\epsilon_+\,, \\
\delta^2_\eta C_i &= \frac{i}{2} H^{-2}\partial_j H\,\bar\epsilon_+\,\gamma^{\underline{i}\,\underline{j}\,\underline{t}}\,\epsilon_+\,, \\
\delta^2_\eta B_{ij} &= -\frac{i}{2} H^{-1}\partial_k H\,\bar\epsilon_+\,\gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{t}}\,\epsilon_+\,, \\
\delta^2_\eta C_{tij} &= \frac{i}{2} H^{-1}\partial_k H\,\bar\epsilon_+\,\gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{t}}\,\epsilon_+\,.
\end{aligned}
\tag{3.13}
$$

These fields again agree precisely with the results of [23, 10]. The cross term g_{ti} in the resulting D0-brane metric generates an angular momentum, J_{ij}, in the same manner as above, and the 1-form C_i generates a magnetic dipole moment, μ_{ij}. They are just the dimensional reduction of the spin $J_{\mu ij}$, as in (2.23). With

$$
J_{tij} = \frac{1}{2\pi R} J_{ij}\,, \qquad J_{zij} = \frac{1}{2\pi R}\mu_{ij}\,,
\tag{3.14}
$$

we have

$$J_{ij} = -\mu_{ij} = -\frac{i}{2} T_0 \, \bar{\epsilon}_+ \, \gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ \, . \tag{3.15}$$

The gyromagnetic ratio, g, of these two moments is given in general via [23]

$$\mu_{ij} = g \frac{Q}{2M} J_{ij} \, , \tag{3.16}$$

for a particle of mass $M = T_0$ and charge Q. Taking care to account for the implicit factor of κ_{10}^2/Ω_8 in our definition of μ_{ij}, relative to [23], we have that $Q = -2T_0$. In other words, $g = 1$ [23]. This is thus a natural consequence of the M-theoretic origin of the polarized D0-brane: both the off-diagonal terms in the metric, and the R-R magnetic potential have a common origin in eleven dimensions.

Finally, there is a magnetic dipole moment, μ_{ijk}, associated with B_{ij} and an electric dipole moment, d_{ijk}, associated with C_{tij}. Both come from the eleven-dimensional dipole moments (2.26). We have

$$\mu_{tijk} = \frac{1}{2\pi R} d_{ijk}, \qquad \mu_{zij} = \frac{1}{2\pi R} \mu_{ijk} \, , \tag{3.17}$$

so that

$$d_{ijk} = -\mu_{ijk} = -\frac{i}{2} T_0 \, \bar{\epsilon}_+ \, \gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{t}} \, \epsilon_+ \, . \tag{3.18}$$

The gyromagnetic ratio of these two ten-dimensional dipole moments is again $g = 1$ [10], and this is again due to the fact that both moments have a common eleven-dimensional origin.

4. On the asymptotic light cone

4.1 A tale of two bases

To discuss the light-like compactification of the bosonic pp-wave and its superpartners, we define the coordinates

$$x^+ = -\frac{1}{2}(t + z) \, , \qquad x^- = t - z \, , \tag{4.1}$$

so that the pp-wave metric (2.4) becomes

$$ds^2 = 2dx^+ dx^- + F dx^{-2} + dx^i dx^i \, , \tag{4.2}$$

where

$$F(r) = H(r) - 1 = \frac{2}{7} \frac{\kappa_{11}^2}{\Omega_8} \frac{P}{r^7} \, , \tag{4.3}$$

ensures asymptotic flatness. Since $F \to 0$ as $r \to \infty$, x^+ and x^- are light cone coordinates only at infinity in the transverse space. In particular, our time coordinate x^+ is related to the light cone time x^+_{LC} via

$$x^+ = x^+_{LC} - \frac{F}{2} x^- \, , \tag{4.4}$$

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and x^- is space-like everywhere except as $r \to \infty$. In terms of the energy-momentum tensor, T^{AB}, the ADM energy-momentum is

$$P^A = \int dx^- d^9x\, T^{+A} , \tag{4.5}$$

which gives the light cone energy $P^- = 0$. The longitudinal light cone momentum is

$$P^+ = \int dx^- P(x^-) , \tag{4.6}$$

and we will again assume that the momentum density, $P(x^-) = \text{const}$.

Perhaps the most natural basis to use in the analysis of the spacetime (4.2) is[3]

$$E^+ = dx^+ + \frac{1}{2} F dx^- , \qquad E^- = dx^- , \qquad E^{\underline{i}} = dx^i , \tag{4.7}$$

so that

$$ds^2 = 2E^+E^- + E^{\underline{i}}E^{\underline{i}} . \tag{4.8}$$

Then the Killing spinors are given by $\epsilon = \epsilon_-$, where $\Gamma^- \epsilon_- = 0$. With the broken supersymmetries $\epsilon = \epsilon_+$, where $\Gamma^+\epsilon_+ = 0$, there is a single non-zero component of the gravitino:

$$\delta_\epsilon \Psi_- = -\frac{1}{4} \partial_i F\, \Gamma^{\underline{i}} \Gamma^- \, \epsilon_+ . \tag{4.9}$$

This is manifestly normalizable — it is null — and satisfies the tracelessness condition $\Gamma^A \delta_\epsilon \Psi_A = 0$, by virtue of the fact that $(\Gamma^-)^2 = 0$. It is interesting to see that these two properties coincide here, when they did not above (nor will they below). A second order supersymmetry transformation, with *no* compensating Lorentz transformations, gives just two corrections to the bosonic fields:

$$\begin{aligned} \delta^2_\epsilon E^{\underline{i}}{}_- &= -\frac{i}{4} \partial_j F\, \bar\epsilon_+ \Gamma^{\underline{i}\,\underline{j}\,-} \epsilon_+ , \\ \delta^2_\epsilon A_{ij-} &= \frac{i}{4} \partial_k F\, \bar\epsilon_+ \Gamma^{\underline{i}\,\underline{j}\,\underline{k}\,-} \epsilon_+ . \end{aligned} \tag{4.10}$$

The 3-form and metric thus have the same asymptotic structure as in section 2.

Despite the simplicity of working in this basis, it is ill-suited to dimensional reduction along x^-. To perform such a compactification, we must rather work with the basis

$$E^{\underline{t}} = F^{-1/2} dx^+, \qquad E^{\underline{z}} = F^{-1/2} dx^+ + F^{1/2} dx^- , \qquad E^{\underline{i}} = dx^i , \tag{4.11}$$

in terms of which the metric (4.2) is again given by (2.12). Such a choice has its own problems, however, the root cause of which seems to be the following. For the basis

[3] We drop the underlines on the $\pm$ tangent space indices in the following. To avoid confusion, note that gamma matrices are written in terms of tangent space components, and the gravitino in terms of curved space components.

(4.7), we have $E^+ \to dx^+$ and $E^- \to dx^-$ as $r \to \infty$, so the metric on the $x^\pm$ plane is $ds^2 \to 2dx^+dx^-$ as expected. On the other hand, for the basis (4.11) we naïvely have $E^{\underline{t}} \to F^{-1/2}dx^+$ and $E^{\underline{z}} \to F^{-1/2}dx^+$, so that $ds^2 \to 0$. The metric is null at infinity. We will attempt to make sense of this below but, in the meantime, let us proceed.

Working with the basis (4.11), the Killing spinors are $\epsilon = F^{-1/4}\epsilon_-$. The natural broken supersymmetry parameters are then $\epsilon = F^{-1/4}\epsilon_+$ where $P_+\epsilon_+ = \epsilon_+$, with P_+ as in (2.17). This choice gives

$$\begin{aligned} \delta_\epsilon \Psi_- &= \frac{1}{2} F^{-3/4} \partial_i F \, \Gamma^{\underline{i}\,\underline{t}} \, \epsilon_+ \, , \\ \delta_\epsilon \Psi_i &= -\frac{1}{2} F^{-5/4} \partial_i F \, \epsilon_+ \, . \end{aligned} \tag{4.12}$$

This gravitino is now *non*-normalizable, however, this being due to the fact that the zero mode parameter $\epsilon = F^{-1/4}\epsilon_+$ blows up at infinity, rather than approaching a constant. The choice of multiplicative factor does not satisfy the condition discussed above *viz*, that it approach 1 as $r \to \infty$. The second order variations

$$\begin{aligned} \delta^2_\epsilon E^{\underline{t}}{}_i &= \frac{i}{16} F^{-3/2} \partial_j F \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ \, , \\ \delta^2_\epsilon E^{\underline{i}}{}_+ &= -\frac{7i}{16} F^{-2} \partial_j F \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ \, , \\ \delta^2_\epsilon E^{\underline{z}}{}_i &= \frac{i}{2} F^{-3/2} \partial_j F \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ \, , \\ \delta^2_\epsilon A_{ij-} &= \frac{i}{2} F^{-1} \partial_k F \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{t}} \epsilon_+ \, , \end{aligned} \tag{4.13}$$

give the modified metric

$$ds^2 = 2dx^+dx^- + F dx^{-2} + dx^i dx^i + \frac{i}{2} F^{-1} \partial_j F \, \bar{\epsilon}_+ \, \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \, \epsilon_+ dx^- dx^i \, , \tag{4.14}$$

in which the problem manifests itself again. That is, the angular momentum and dipole moments as given by (2.22) and (2.25) respectively, are no longer defined; we have $F^{-1} \to r^7/k$ as $r \to \infty$, instead of $H^{-1} \to 1$, so the corrections to the bosonic fields do not fall off fast enough at infinity. On the face of it, we should simply choose a different gauge for the fermionic zero modes. However, it would seem that these problems are due rather to our choice of basis. After all, the basis (4.11) naïvely gives a null metric at infinity, rather than a flat one (in light cone coordinates). This, in turn, is related to the fact that compactification of x^- gives the $r \to 0$ "near-horizon" limit of the D0-brane solution, which is no longer asymptotically flat. Quantities such as angular momentum and dipole moments are potentially ill-defined in such spacetimes.

4.2 Dimensional reduction on the asymptotic light cone

We are interested here in the dimensional reduction along x^- of the pp-wave and its superpartners of the previous subsection. Such a light-like compactification of M-theory is related to finite N Matrix theory [6, 7, 8]. Now in the standard reduction described in section 3, we identified $z \sim z + 2\pi R_s$, but the proper circumference, L_s of the space-like circle is

$$L_s = H^{1/2} 2\pi R_s\,, \tag{4.15}$$

so that $L_s(r \to \infty) = 2\pi R_s$. However, if we identify $x^- \sim x^- + 2\pi R_l$, the proper circumference, L_l, of the light-like circle is

$$L_l = F^{1/2} 2\pi R_l\,, \tag{4.16}$$

and so $L_l(r \to \infty) = 0$. The compact direction is thus only *asymptotically* light-like. That our basis (4.11) gives a null metric at infinity should perhaps then be expected.

The two compactifications are related [7]: in the limit $R_s \to 0$, the space-like compactification is related to the light-like one by an infinite boost along z, with velocity

$$v = \frac{R_l}{\sqrt{R_l^2 + 4R_s^2}} \approx 1 - 2\frac{R_s^2}{R_l^2}\,. \tag{4.17}$$

Indeed, the two metrics (2.4) and (4.2) are related by just such a boost, provided that we replace the original momentum density $P = 1/(2\pi R_s) N/R_s$ with $P = 1/(2\pi R_l) N/R_l$. More specifically, the light-like compactification of M-theory, with gravitons carrying momentum $P^+ = N/R_l$ around the light-like circle, is related by the boost (4.17) to M-theory on the space-like circle, in the limits $R_s \to 0$, $\alpha' \to 0$ and $U = R_s/\alpha' =$ fixed [7, 8]. This is just the Maldacena decoupling limit [38] applied to D0-branes, as considered in [39, 40]. Supergravity in this background is then dual to Matrix theory at finite N.

There are some subtleties, however. Firstly, as we have already seen, the compactification is only asymptotically light-like [41, 42]. More importantly, one should be careful about the regimes of validity of the different descriptions, the key point being that the radius of the eleventh dimension is set by the dilaton [41]. The story is explained in detail in [42]: for large distances $r > l_p N^{1/3}$ (the UV of the gauge theory), the effective coupling constant of the gauge theory is small, so that Matrix perturbation theory is valid. As r decreases (moving toward the IR), the Matrix theory becomes strongly coupled. For $l_p N^{1/7} < r < l_p N^{1/3}$, supergravity in the $r \to 0$ limit of the D0-brane solution is the better description, since the spacetime curvature is small compared to the string scale, and the string coupling $e^\phi \ll 1$. For smaller distances still, this solution becomes eleven-dimensional, since the dilaton is blowing up with the effective gauge theory coupling. The relevant description, for $l_p N^{1/9} < r < l_p N^{1/7}$, is that of a pp-wave on an asymptotically light-like circle, which is what we consider here. This description in turn breaks down at $r = l_p N^{1/9}$, at which point the radius of the eleventh dimension is itself equal to r, and the boosted black string of section 2 becomes unstable [39].

To compactify along a light-like circle, one considers the limit of a compactification on a space-like circle, which is almost light-like [7, 43]. In other words, one "regularizes" the original metric such that the coordinate in question is everywhere space-like, performs the standard compactification of the space-like coordinate, and only then does one take the light-like limit [43]. In the case at hand, the function F acts as just such a regulator [44], except as $r \to \infty$. To ensure that x^- is *everywhere* space-like, we thus write

$$F = \frac{R_s}{R_l} + \frac{k}{r^7}, \tag{4.18}$$

perform the compactification in the standard way, and then take the limit $R_s \to 0$. This procedure is thus essentially equivalent to the compactification along z as above, but with the 1 finally dropped from the harmonic function H [45, 46, 47]. Indeed, it has been shown explicitly in [45, 9] that compactification along x^- of the pp-wave (4.2) does indeed give the short distance, or "near-horizon", limit of the D0-brane solution (3.6). Moreover, the null reduction of the action describing a massless particle in eleven dimensions gives that describing a non-relativistic D0-brane [9], which is precisely the Matrix theory action, up to the $U(N)$ gauge symmetry. We should also note that the strange behaviour of the basis (4.11) is cured via this regularization of the function F. With F as in (4.18), we have $E^{\underline{t}} \to \sqrt{R_l/R_s}dx^+$ and $E^{\underline{z}} \to \sqrt{R_l/R_s}dx^+ + \sqrt{R_s/R_l}dx^-$, so that $ds^2 \to 2dx^+dx^-$ as required.

Now we can see why the gravitino of the previous subsection is not normalizable, and why the problems in defining the spin and dipole moments occur. With F as in (4.18), the norm of the gravitino (4.12) is

$$|\delta_\epsilon \Psi|^2 = \frac{1}{2}\frac{R_l}{R_s} P^+ \epsilon_+^\dagger \epsilon_+ , \tag{4.19}$$

where $P^+ = N/R_l$. The infinite norm is then due to taking the light-like limit $R_s \to 0$. But this is precisely what we would find in the space-like compactification on a circle of vanishing radius: with $P^z = N/R_s$, the norm of the gravitino in (2.19) is also infinite in the limit $R_s \to 0$. As the latter is Lorentz-equivalent to the former, the fact that the gravitino (4.12) of the previous subsection is non-normalizable should not come as any surprise!

Similar comments can be made concerning both the spin, J_{-ij}, and dipole moment, μ_{-ijk} generated by the second order variations (4.13). Since we now have $F \to R_s/R_l$ as $r \to 0$, these physical quantities are well-defined, since the corrections to the bosonic fields then fall off as $1/r^8$ as required. However, both quantities are then formally infinite in the $R_s \to 0$ limit, since they go like $1/R_s$:

$$J_{-ij} = \frac{i}{2} P^+ \frac{R_l}{R_s}\, \bar{\epsilon}_+ \Gamma^{\underline{i}\underline{j}\underline{t}} \epsilon_+ ,$$

$$\mu_{-ijk} = \frac{i}{2} P^+ \frac{R_l}{R_s} \bar{\epsilon}_+ \Gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{t}} \epsilon_+ \,. \tag{4.20}$$

But this is just what we would expect upon taking the $R_s \to 0$ limit of the corresponding quantities in subsection 2.2.

4.3 D0-branes as $r \to 0$, and their superpartners

Dimensional reduction along x^- is now straightforward. With the Kaluza-Klein ansätze as in (3.1) and (3.3), but with z replaced by $-$, the purely bosonic solution (4.2) reduces to

$$\begin{aligned} ds^2 &= -F^{-7/8} dx^{+2} + F^{1/8} dx^i dx^i \,, \\ C_+ &= F^{-1} \,, \\ e^{2\phi} &= F^{3/2} \,, \end{aligned} \tag{4.21}$$

where now $F = k/r^7$. This is precisely the decoupling limit of the D0-brane solution (3.6), as in [45, 9], and a conformal transformation to the "dual" frame takes the metric to that of $\mathrm{AdS}_2 \times S^8$ [40]. One might be concerned that the $r \to 0$ limit of the R-R 1-form potential in (3.6) should give a constant, and not $C_+ = F^{-1}$. However, one should instead consider the corresponding limit of the field strength $F_{rt} = \partial_r(H^{-1}) \to F_{r+} = 7r^6/k$, for which the 1-form above *is* the corresponding potential. As to the ten-dimensional superpartners, with the reduction formulae (3.8)–(3.11) we find the fermions

$$\begin{aligned} \delta_\eta \lambda &= \frac{3\sqrt{2}}{8} F^{-41/32} \partial_i F \, \gamma^{\underline{i}\,\underline{t}} \, \epsilon_+ \,, \\ \delta_\eta \psi_+ &= -\frac{7}{16} F^{-55/32} \partial_i H \, \gamma^{\underline{i}\,\underline{t}} \, \epsilon_+ \,, \\ \delta_\eta \psi_i &= \frac{1}{16} F^{-39/32} \partial_j F (\gamma^{\underline{i}\,\underline{j}} - 7\delta^{ij}) \epsilon_+ \,, \end{aligned} \tag{4.22}$$

In other words, the first order supersymmetry variations of the solution (4.21) are precisely the $r \to 0$ limits of the first order variations, (3.12), of the D0-brane. At this order, taking the $r \to 0$ limit of the D0-brane induces a corresponding limit in the entire supermultiplet.

This makes sense from the ten-dimensional perspective. Replacing H with F in the purely bosonic D0-brane solution just gives a non-asymptotically flat solution of type-IIA supergravity. It still preserves 1/2 of the supersymmetries, the Killing spinors now being $\eta = F^{-7/32} \epsilon_-$. Taking the broken supersymmetry parameters to be $\eta = F^{-7/32} \epsilon_+$, we do indeed find the above fermions. One might object that this choice is invalid since it does not approach a constant at infinity. But then the solution is not asymptotically flat, so quantities such as the supercharge are ill-defined.

Up to exchanging H for F, the second order variations are unchanged relative to (3.13):

$$\begin{aligned}
\delta_\eta^2 e^{\underline{t}}{}_i &= \frac{i}{16} F^{-23/16} \partial_j F\, \bar{\epsilon}_+ \gamma^{\underline{i}\,\underline{j}\,\underline{t}} \epsilon_+ \,, \\
\delta_\eta^2 e^{\underline{i}}{}_+ &= -\frac{7i}{16} F^{-31/16} \partial_j F\, \bar{\epsilon}_+ \gamma^{\underline{i}\,\underline{j}\,\underline{t}} \epsilon_+ \,, \\
\delta_\eta^2 C_i &= \frac{i}{2} F^{-2} \partial_j F\, \bar{\epsilon}_+ \gamma^{\underline{i}\,\underline{j}\,\underline{t}} \epsilon_+ \,, \\
\delta_\eta^2 B_{ij} &= -\frac{i}{2} F^{-1} \partial_k F\, \bar{\epsilon}_+ \gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{t}} \epsilon_+ \,,
\end{aligned} \tag{4.23}$$

although there is no electric R-R 3-form potential. It is straightforward to compute these corrections to the solution (4.21) within type-IIA supergravity directly, and we do indeed find that $\delta_\eta^2 C_{+ij} = 0$. This is due to the fact that $C_+ = F^{-1} \neq F^{-1} - 1$ in the purely bosonic solution. In the D0-brane solution, the -1 is a pure gauge term which ensures that the potential vanishes at infinity. In this case, since the spacetime is not asymptotically flat, there is no such requirement on the behaviour of C_+. Of course, the absence of the R-R 3-form is correlated with the fact that there is no electric dipole moment for this field: upon dimensional reduction, μ_{-ijk} in (4.20) gives a magnetic dipole moment associated with B_{ij} only. In a similar manner, J_{-ij} gives a magnetic dipole moment associated with C_i, but no ten-dimensional spin (despite the fact that there is a corresponding cross-term in the metric). In other words, the near-horizon solution of the D0-brane has no spin, and no D2-brane dipole moment. It has magnetic dipole moments alone.

5. Discussion

We have computed the superpartners of the purely bosonic eleven-dimensional pp-wave in two coordinate systems, one adapted to a space-like dimensional reduction along z, the other adapted to a light-like dimensional reduction along x^-. The former gives the bosonic D0-brane and its superpartners, the latter gives the $r \to 0$ limit of these solutions. This "near-horizon" limit of the D0-brane is a non-asymptotically flat spacetime, so its physical ADM-like properties are ill-defined. This is reflected in the fact that the eleven-dimensional basis suited to the light-like dimensional reduction must be regularized in order for the compactification to be carried out, and so that the dipole moments of the supergraviton spacetimes can be defined. Even then, the first order gravitino as well as these dipole moments are formally infinite, but we have argued that this is to be expected. The light-like compactification is Lorentz-equivalent to the spacelike compactification on a circle of vanishing radius and, in this limit, certain quantities do indeed blow up. We have shown that the superpartners of the $r \to 0$ limit of the D0-brane possess neither spin nor D2-brane electric dipole moments. Magnetic dipole moments, with respect to the R-R 1-form and NS-NS 2-form, do persist however. It would be interesting to understand these results from the perspective of Matrix theory.

Of course, one could further consider higher order supersymmetry transformations to determine, for example, the quadrupole moments of the eleven-dimensional supergraviton. Via dimensional reduction, this would be a simpler way of determining the higher order variations of the D0-brane solution, rather than working with type-IIA supergravity directly. However, the necessity of compensating Lorentz transformations in eleven dimensions ensures that such a calculation would still be rather tedious.

The couplings of the background supergravity fields to the "worldvolume" fermions of the eleven-dimensional supergraviton could also be determined. And this would allow for the analysis of interactions between supergravitons, via the consideration of a probe action for a massless spinning particle in a general eleven-dimensional background. Indeed, using techniques similar to those of [10], this procedure has been carried out in [48], and the resulting couplings have a form similar to that for the D0-brane in (1.1). However, only the first two terms of (1.1), coming from the eleven-dimensional coupling $\partial_j h_{+i}\bar{\theta}\gamma^{ij}\theta$, were found in [48]. We have shown that there should be further couplings of the pp-wave to the background 3-form, and it would be of interest to determine these. Of course, they should be of the form $\partial_k A_{+ij}\bar{\theta}\gamma^{ijk}\theta$ [10], but the explicit computation would provide a nice consistency check.

Acknowledgments

We thank Mark van Raamsdonk for pointing out a misprint in eq. (12) of [10]. D.B. thanks Carlos Herdeiro, Bert Janssen, Simon Ross and Paul Saffin for useful conversations and comments on early drafts of this paper, and is supported in part by the EPSRC grant GR/N34840/01. A.C. thanks Wati Taylor for numerous useful conversations, and is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818.

A. Conventions

We use the signature $(-+\ldots+)$. Eleven-dimensional coordinates $x^A, A, B{=}0,\ldots,10$ are written in terms of ten-dimensional coordinates $x^a, a, b = 0,\ldots,9$ and z (or x^-). The ten-dimensional coordinates are written in terms of t (or x^+) and $x^i, i, j = 1,\ldots,9$. Tangent space directions are denoted by an underline. The eleven-dimensional Planck length $l_p = g_s^{1/3}\sqrt{\alpha'}$, so that $\kappa_{11}^2 = 2\pi R g_s^2 \kappa_{10}^2$ where g_s is the string coupling constant and the radius of the eleventh dimension is $R = g_s\sqrt{\alpha'}$.

The Dirac matrices satisfy

$$\{\Gamma^{\underline{A}}, \Gamma^{\underline{B}}\} = 2\eta^{\underline{A}\,\underline{B}}\,. \tag{A.1}$$

Explicitly, the 32×32 component Dirac matrices are given by

$$\Gamma^{\underline{i}} = \begin{pmatrix} 0 & \sigma^{\underline{i}} \\ \sigma^{\underline{i}} & 0 \end{pmatrix}, \qquad \Gamma^{\underline{t}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \Gamma^{\underline{z}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.2}$$

where $\sigma^{\underline{i}}$ denote 16×16 real, symmetric SO(9) Dirac matrices. Eleven-dimensional spinors are anticommuting and Majorana; they obey the useful identities

$$\bar{\psi}\, \Gamma^{\underline{i}_1} \cdots \Gamma^{\underline{i}_n} \Gamma^{\underline{t}} \lambda = -\bar{\lambda}\, \Gamma^{\underline{i}_n} \cdots \Gamma^{\underline{i}_1} \Gamma^{\underline{t}} \psi\,, \tag{A.3}$$

$$\bar{\psi}\, \Gamma^{\underline{i}_1} \ldots \Gamma^{\underline{i}_n} \lambda = (-)^n \bar{\lambda}\;\, \Gamma^{\underline{i}_n} \ldots \Gamma^{\underline{i}_1} \psi\,. \tag{A.4}$$

Such a spinor can be split as $\epsilon = P_+\epsilon + P_-\epsilon \equiv \epsilon_+ + \epsilon_-$, where the projection operators

$$P_\pm = \frac{1}{2}\left(1 \pm \Gamma^{\underline{t}}\Gamma^{\underline{z}}\right) = \frac{1}{2}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}. \tag{A.5}$$

In terms of a 16-component spinor ϵ,

$$\epsilon_\pm = \begin{pmatrix} \epsilon \\ \pm\epsilon \end{pmatrix}. \tag{A.6}$$

The zero mode fermions in the text are given in terms of ϵ_+, and the only non-vanishing fermion bilinears that can be constructed with this are

$$\bar{\epsilon}_+ \Gamma^{\underline{i}\,\underline{j}\,\underline{t}} \epsilon_+\,, \qquad \bar{\epsilon}_+ \Gamma^{\underline{i}\,\underline{j}\,\underline{k}\,\underline{t}} \epsilon_+\,. \tag{A.7}$$

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